Equivalent Stress

Von Mises Stress

Solid Mechanics I – Understanding the Physics

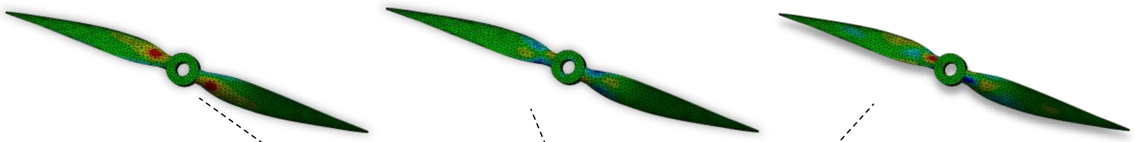
Stress and Local Equilibrium

Ansys

Equivalent Stress

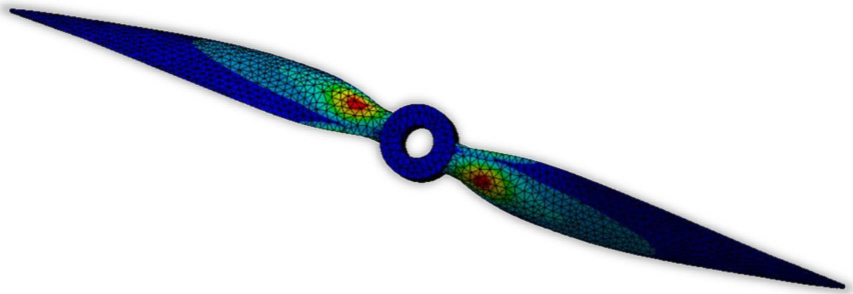
We know that a stress state can be comprehensively described by a 3x3 tensor. Is there a way that we can use one value to represent the state of a material? Equivalent stress is defined to serve this purpose.

- In evaluation of stress results, we need 9 contour plots to visualize stress tensor
- Equivalent stress allows one to view stress of the structure by one plot
- Equivalent stress can be used as a scalar indicator to determine material failure



$$\begin{bmatrix} \boxed{\sigma_{xx}} & \boxed{\sigma_{xy}} & \boxed{\sigma_{xz}} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

9 stress components



Equivalent stress, one value

Equivalent Stress: von Mises Stress

Here we are introducing one widely used equivalent stress, the von-Mises stress σ_v.

- Calculate von Mises stress from stress components

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\sigma_v = \sqrt{\frac{(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{yy}-\sigma_{zz})^2 + (\sigma_{zz}-\sigma_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2)}{2}}$$

- Calculate von Mises stress from principal stress

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\sigma_v = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2)}{2}}$$

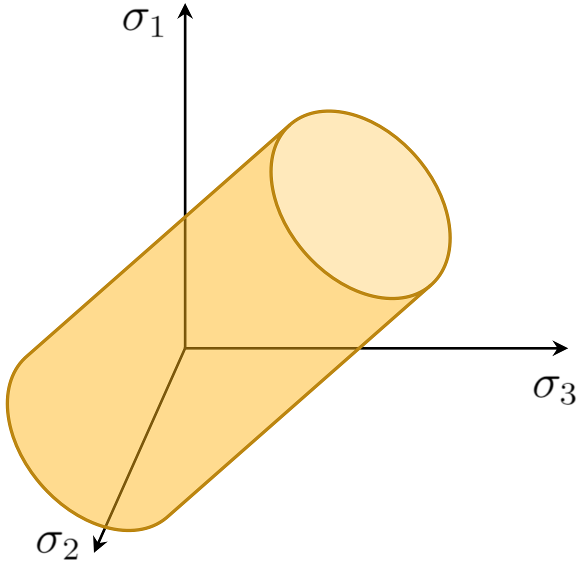
Note that the two expressions give the same equivalent stress value for a stress status

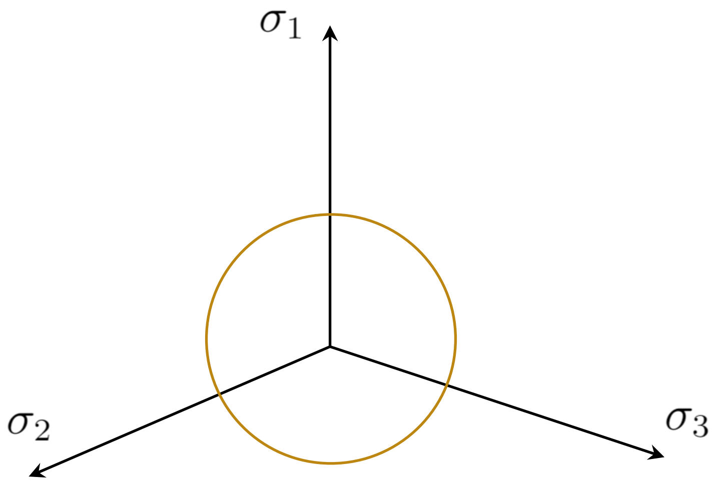
Equivalent Stress: von Mises Stress

Von Mises stress is best explain in principal coordinate system, because of easier visualization.

$$\sigma_v = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)}{2}}$$

- In the three brackets, the terms are the differences between the three principal stresses
- This expression can be plotted as a cylindrical surface in the principal coordinate system





The cylindrical surface is a perfect circle in octahedral plane

Equivalent Stress: von Mises Stress

On one von Mise surface, all the points have the same von Mises stress value.

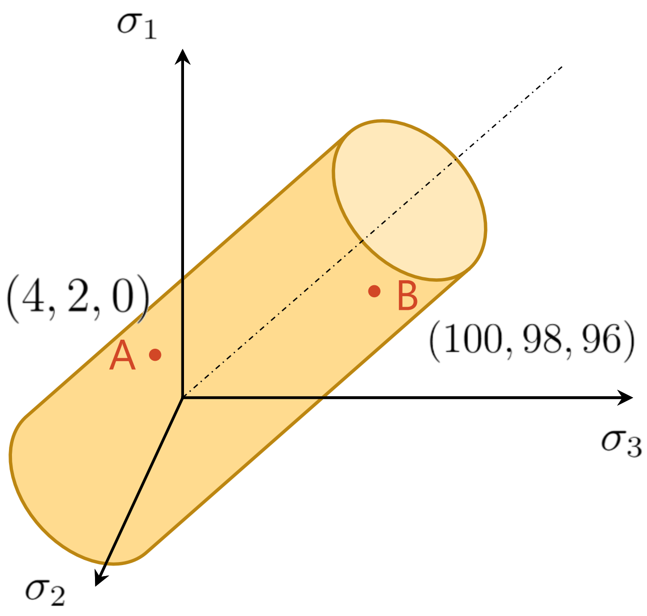
- Point A, principal stress state is $\begin{bmatrix} 100 & & \\ & 98 & \\ & & 96 \end{bmatrix}$

$$\sigma_v = \sqrt{\frac{(100-98)^2 + (98-96)^2 + (96-100)^2)}{2}} = 3.46$$

- Point B, principal stress state is $\begin{bmatrix} 4 & & \\ & 2 & \\ & & 0 \end{bmatrix}$

$$\sigma_v = \sqrt{\frac{(4-2)^2 + (2-0)^2 + (0-2)^2)}{2}} = 3.46$$

$$\sigma_v = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2)}{2}}$$

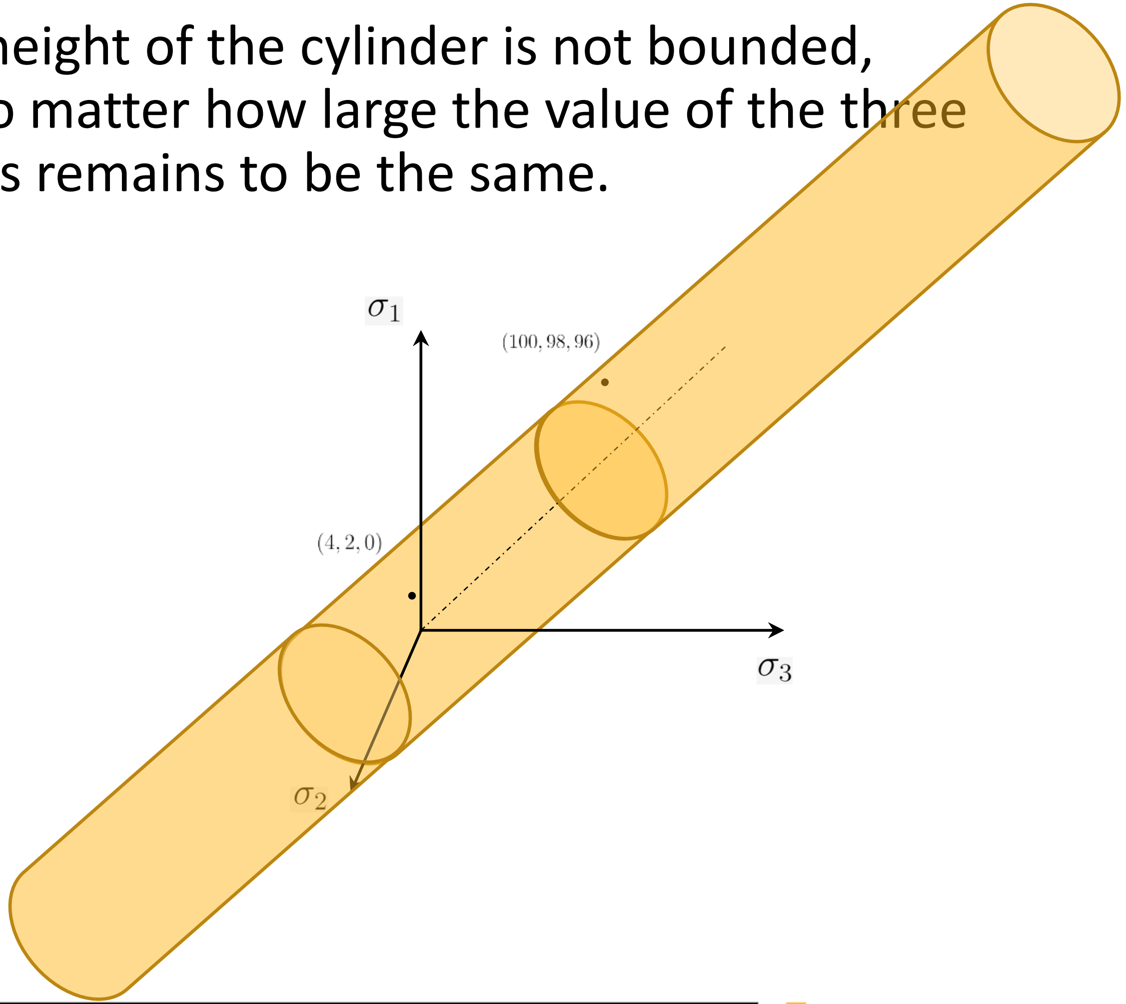


Note that the difference between principal stresses are the same for point A and B

Equivalent Stress: von Mises Stress

Another important feature of von Mises surface, the height of the cylinder is not bounded, meaning the two ends extend infinitely. This means no matter how large the value of the three principal stresses are, the von Mises stress could stress remains to be the same.

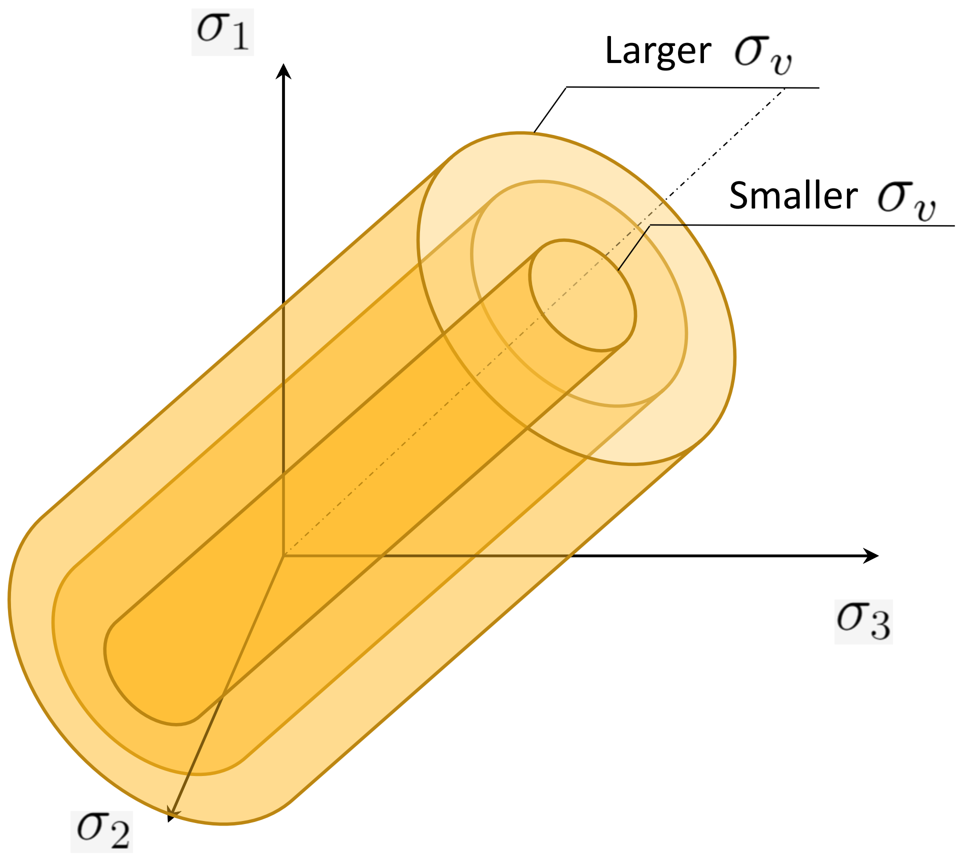
$$\sigma_v = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)}{2}}$$



Equivalent Stress: von Mises Stress

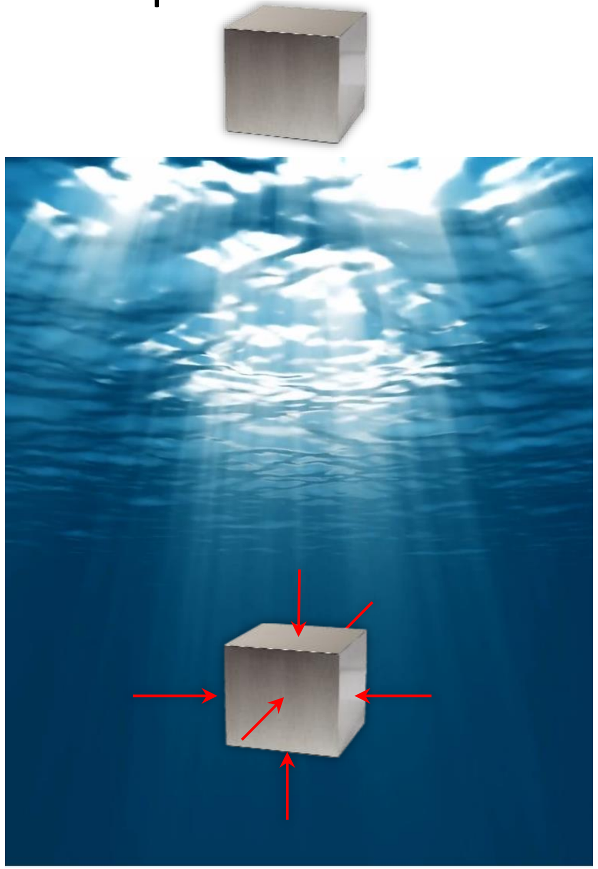
For a larger von Mises stress value, the radius of the cylindrical will expand, and for a smaller von Mises stress value, the radius of the cylindrical surface will shrink

$$\sigma_v = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)}{2}}$$



Equivalent Stress: von Mises Stress

If we throw a small steel cube to deep sea. The six faces of the steel cube are under pressure from water. And the pressure grows with the depth of water. But the von Mises stress remains to be the same for the entire sinking process.

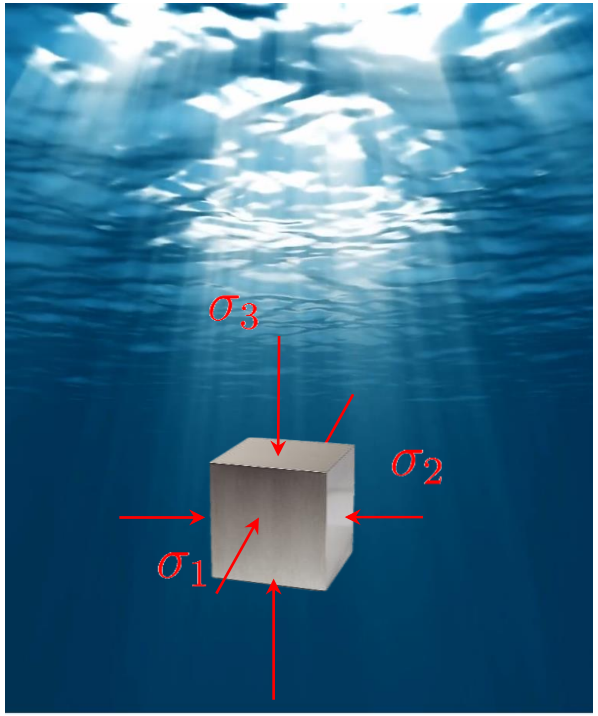


Equivalent Stress: von Mises Stress

The differences between the three principal stresses remain to be the same while sinking in the water

- The principal stresses over the faces of the cube are pressure from Buoyancy force.
- Let's ignore the change of Buoyancy force caused by the dimension of the cube since compared to the water depth, it's very small.
- The value of the stresses grows proportionally while sinking, the differences between three principal stresses remain the same

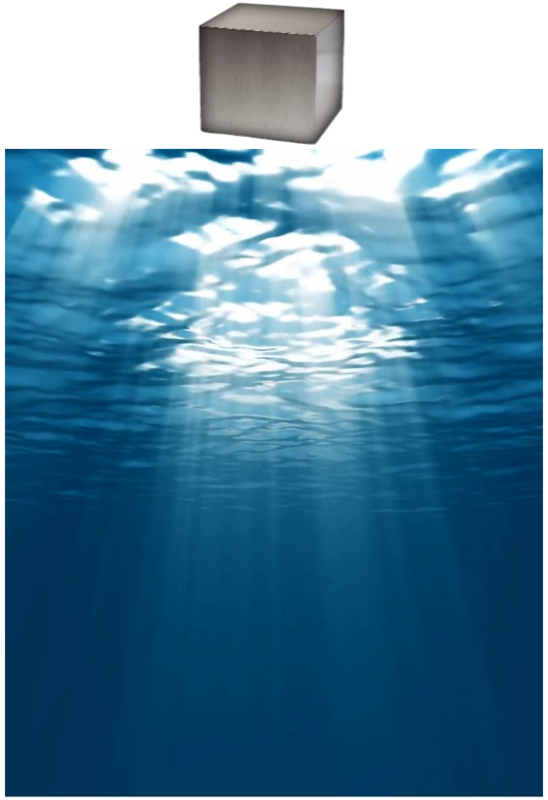
$$\sigma_v = \sqrt{\frac{(\overbrace{\sigma_1 - \sigma_2}^{0})^2 + (\overbrace{\sigma_2 - \sigma_3}^{0})^2 + (\overbrace{\sigma_3 - \sigma_1}^{0})^2)}{2}}$$

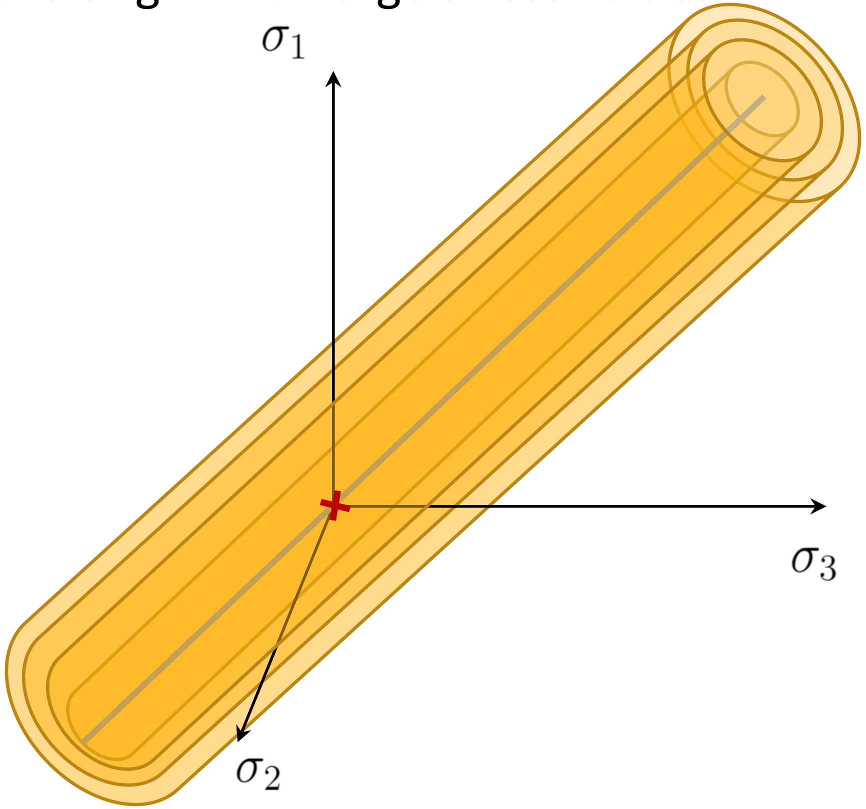


Note that only the differences between the three principal stress affect the value of von Mises stress. This is why von Mises is a good equivalent stress to represent the distortion of a material.

Equivalent Stress: von Mises Stress

If we plot the stress state on the principal coordinate system, we will see that cylindrical surface for 0 von Mises is minimized to be just the hydrostatic axis. And the point of the stress state slides from origin with principal stresses are all 0 to a location far from the origin with large stress values.





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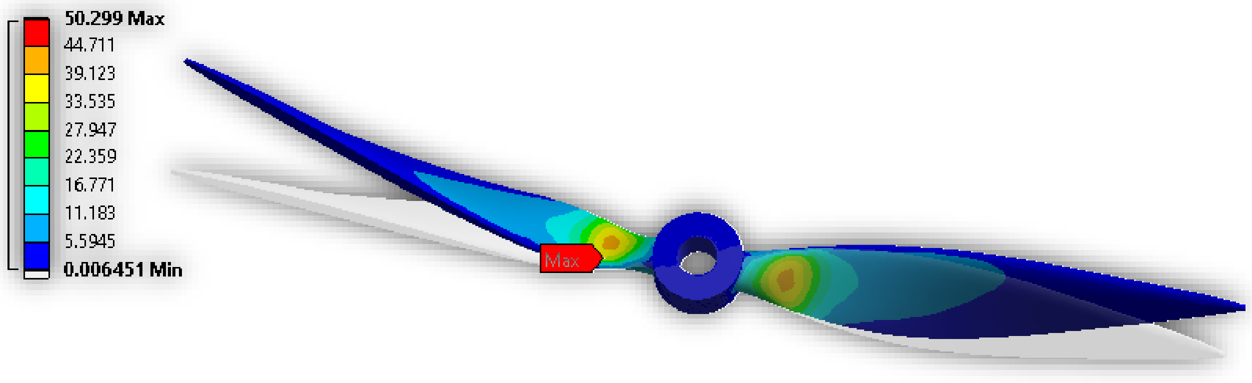
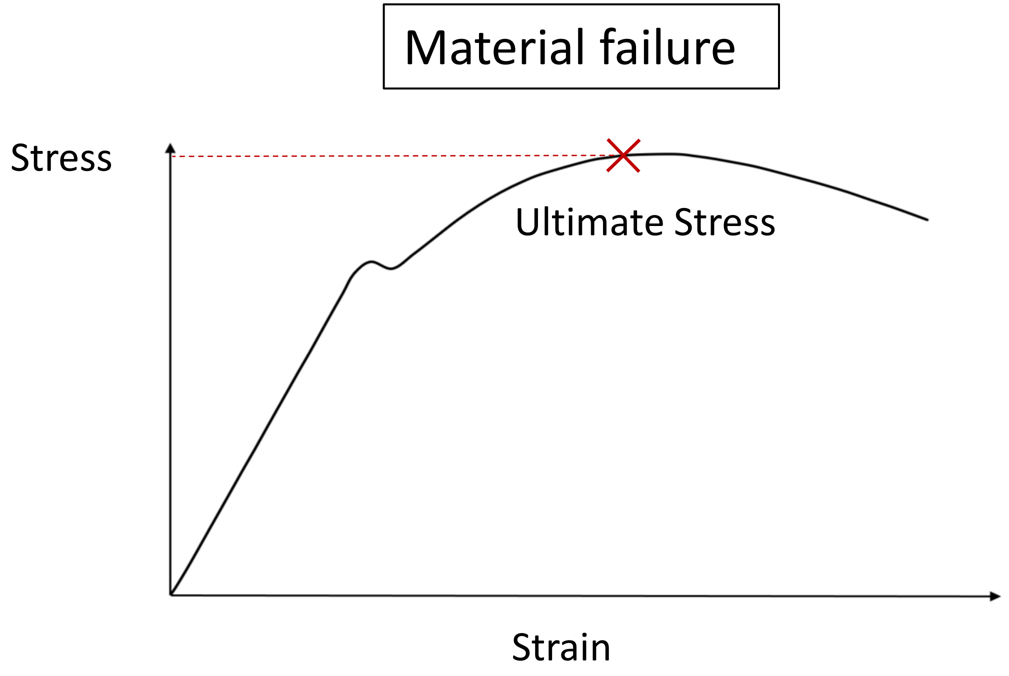
Equivalent Stress: von Mises Stress

Equivalent stress is widely used to represent a material's status for ductile material. Engineers use this simple scalar value to determine if the material has yield or failed.





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